SAT Math Facts & Formulas

Numbers, Sequences, Factors

Integers: . . . , -3, -2, -1, 0, 1, 2, 3, . . .
Reals: integers plus fractions, decimals, and irrationals ($\sqrt{2}$, $\sqrt{3}$, π, etc.)

Order Of Operations: PEMDAS
(Parentheses / Exponents / Multiply / Divide / Add / Subtract)

Arithmetic Sequences: each term is equal to the previous term plus d

Sequence: $t_1, t_1 + d, t_1 + 2d, \ldots$
The n$^{\text{th}}$ term is $t_n = t_1 + (n-1)d$
Number of integers from i_n to $i_m = i_m - i_n + 1$
Sum of n terms $S_n = (n/2) \cdot (t_1 + t_n)$ (optional)

Geometric Sequences: each term is equal to the previous term times r

Sequence: $t_1, t_1 \cdot r, t_1 \cdot r^2, \ldots$
The n$^{\text{th}}$ term is $t_n = t_1 \cdot r^{n-1}$
Sum of n terms $S_n = t_1 \cdot (r^n - 1)/(r-1)$ (optional)

Prime Factorization: break up a number into prime factors (2, 3, 5, 7, 11, . . .)

$$200 = 4 \times 50 = 2 \times 2 \times 2 \times 5 \times 5$$
$$52 = 2 \times 26 = 2 \times 2 \times 13$$

Greatest Common Factor: multiply common prime factors

$$\begin{aligned} 200 &= 2 \times 2 \times 2 \times 5 \times 5 \\ 60 &= 2 \times 2 \times 3 \times 5 \\ \text{GCF}(200, 60) &= 2 \times 2 \qquad \times 5 = 20 \end{aligned}$$

Least Common Multiple: check multiples of the largest number

$$\text{LCM}(200, 60)\text{: } 200 \text{ (no)}, 400 \text{ (no)}, 600 \text{ (yes!)}$$

Percentages: use the following formula to find part, whole, or percent

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

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Averages, Counting, Statistics, Probability

$$\text{average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{sum} = \text{average} \times (\text{number of terms})$$

$$\text{mode} = \text{value in the list that appears most often}$$

$$\text{median} = \text{middle value in the list (which } \textit{must} \text{ be sorted)}$$

Example: median of $\{3, 10, 9, 27, 50\} = 10$
Example: median of $\{3, 9, 10, 27\} = (9 + 10)/2 = 9.5$

Fundamental Counting Principle:

If an event can happen in N ways, and another, independent event can happen in M ways, then both events together can happen in $N \times M$ ways. (Extend this for three or more: $N_1 \times N_2 \times N_3 \ldots$)

Permutations and Combinations (Optional):

The number of permutations of n things taken r at a time is ${}_nP_r = n!/(n-r)!$

The number of combinations of n things taken r at a time is ${}_nC_r = n!/\big((n-r)!\,r!\big)$

Probability:

$$\text{probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$$

The probability of two different events A and B *both* happening is $P(A \text{ and } B) = P(A) \cdot P(B)$, as long as the events are independent (not mutually exclusive).

Powers, Exponents, Roots

$$x^a \cdot x^b = x^{a+b} \qquad x^a/x^b = x^{a-b} \qquad 1/x^b = x^{-b}$$

$$(x^a)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1 \qquad \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

$$\text{If } 0 < x < 1, \text{ then } 0 < x^3 < x^2 < x < \sqrt{x} < \sqrt[3]{x} < 1.$$

Factoring, Solving

$$(x+a)(x+b) = x^2 + (b+a)x + ab \qquad \text{“FOIL”}$$

$$a^2 - b^2 = (a+b)(a-b) \qquad \text{“Difference Of Squares”}$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$
$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$x^2 + (b+a)x + ab = (x+a)(x+b) \qquad \text{“Reverse FOIL”}$$

You can use Reverse FOIL to factor a polynomial by thinking about two numbers a and b which add to the number in front of the x, and which multiply to give the constant. For example, to factor $x^2 + 5x + 6$, the numbers add to 5 and multiply to 6, i.e., $a = 2$ and $b = 3$, so that $x^2 + 5x + 6 = (x+2)(x+3)$.

To solve a quadratic such as $x^2+bx+c = 0$, first factor the left side to get $(x+a)(x+b) = 0$, then set each part in parentheses equal to zero. For example, $x^2+4x+3 = (x+3)(x+1) = 0$ so that $x = -3$ or $x = -1$.

To solve two linear equations in x and y: use the first equation to substitute for a variable in the second. E.g., suppose $x + y = 3$ and $4x - y = 2$. The first equation gives $y = 3 - x$, so the second equation becomes $4x - (3 - x) = 2 \Rightarrow 5x - 3 = 2 \Rightarrow x = 1, y = 2$.

Functions

A function is a rule to go from one number (x) to another number (y), usually written

$$y = f(x).$$

For any given value of x, there can only be one corresponding value y. If $y = kx$ for some number k (example: $f(x) = 0.5 \cdot x$), then y is said to be *directly proportional* to x. If $y = k/x$ (example: $f(x) = 5/x$), then y is said to be *inversely proportional* to x.

The graph of $y = f(x-h) + k$ is the *translation* of the graph of $y = f(x)$ by (h, k) units in the plane. For example, $y = f(x+3)$ shifts the graph of $f(x)$ by 3 units to the left.

Absolute value:

$$|x| = \begin{cases} +x, & \text{if } x \geq 0; \\ -x, & \text{if } x < 0. \end{cases}$$

$$|x| < n \quad \Rightarrow \quad -n < x < n$$
$$|x| > n \quad \Rightarrow \quad x < -n \quad \text{or} \quad x > n$$

Parabolas:

A parabola parallel to the y-axis is given by

$$y = ax^2 + bx + c.$$

If $a > 0$, the parabola opens up. If $a < 0$, the parabola opens down. The y-intercept is c, and the x-coordinate of the vertex is $x = -b/2a$.

Lines (Linear Functions)

Consider the line that goes through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Distance from A to B: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

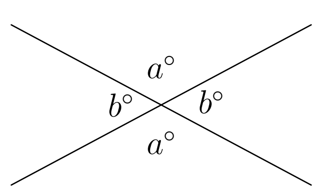
Mid-point of the segment $\overline{AB}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope of the line: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$

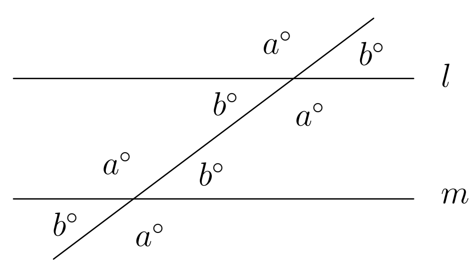
Point-slope form: given the slope m and a point (x_1, y_1) on the line, the equation of the line is $(y - y_1) = m(x - x_1)$.

Slope-intercept form: given the slope m and the y-intercept b, then the equation of the line is $y = mx + b$.

Parallel lines have equal slopes. Perpendicular lines (i.e., those that make a 90° angle where they intersect) have negative reciprocal slopes: $m_1 \cdot m_2 = -1$.



Intersecting Lines



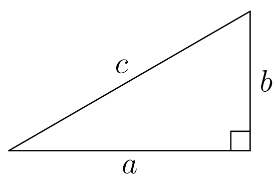
Parallel Lines ($l \parallel m$)

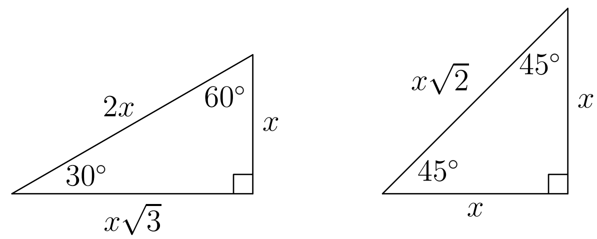
Intersecting lines: opposite angles are equal. Also, each pair of angles along the same line add to 180°. In the figure above, $a + b = 180°$.

Parallel lines: eight angles are formed when a line crosses two parallel lines. The four big angles (a) are equal, and the four small angles (b) are equal.

Triangles

Right triangles:



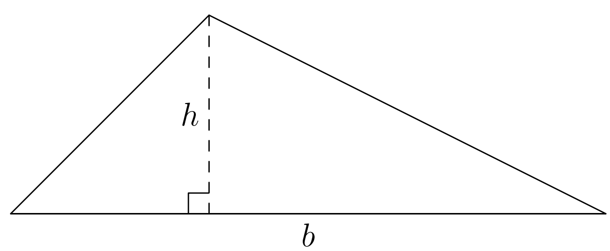


$a^2 + b^2 = c^2$

Special Right Triangles

A good example of a right triangle is one with $a = 3$, $b = 4$, and $c = 5$, also called a 3–4–5 right triangle. Note that multiples of these numbers are also right triangles. For example, if you multiply these numbers by 2, you get $a = 6$, $b = 8$, and $c = 10$ (6–8–10), which is also a right triangle.

All triangles:



$$\text{Area} = \frac{1}{2} \cdot b \cdot h$$

Angles on the inside of any triangle add up to 180°.

The length of one side of any triangle is always *less* than the sum and *more* than the difference of the lengths of the other two sides.

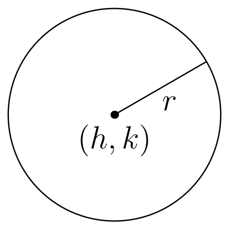
An exterior angle of any triangle is equal to the sum of the two remote interior angles.

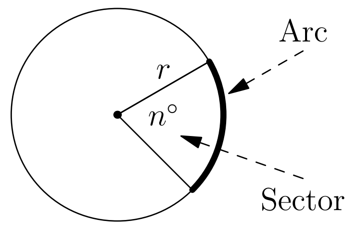
Other important triangles:

Equilateral:	These triangles have three equal sides, and all three angles are 60°.
Isosceles:	An isosceles triangle has two equal sides. The "base" angles (the ones opposite the two sides) are equal (see the 45° triangle above).
Similar:	Two or more triangles are similar if they have the same shape. The corresponding angles are equal, and the corresponding sides are in proportion. For example, the 3–4–5 triangle and the 6–8–10 triangle from before are similar since their sides are in a ratio of 2 to 1.

SAT Math Facts & Formulas

Circles





$\text{Area} = \pi r^2$

$\text{Circumference} = 2\pi r$

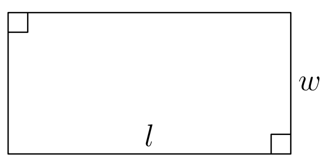
$\text{Full circle} = 360°$

$\text{Length Of Arc} = (n°/360°) \cdot 2\pi r$

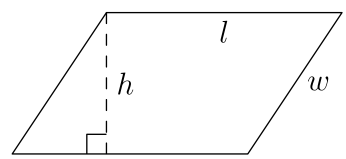
$\text{Area Of Sector} = (n°/360°) \cdot \pi r^2$

Equation of the circle (above left figure): $(x - h)^2 + (y - k)^2 = r^2$.

Rectangles And Friends



Rectangle
(Square if $l = w$)
Area $= lw$

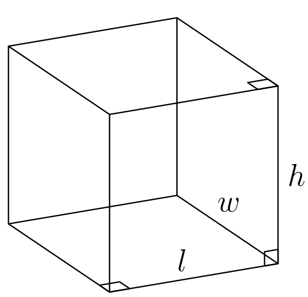


Parallelogram
(Rhombus if $l = w$)
Area $= lh$

Regular polygons are n-sided figures with all sides equal and all angles equal.

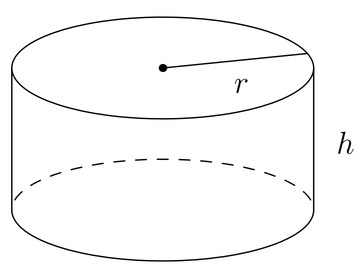
The sum of the inside angles of an n-sided regular polygon is $(n - 2) \cdot 180°$.

Solids



Rectangular Solid

Volume $= lwh$
Area $= 2(lw + wh + lh)$



Right Cylinder

Volume $= \pi r^2 h$
Area $= 2\pi r(r + h)$